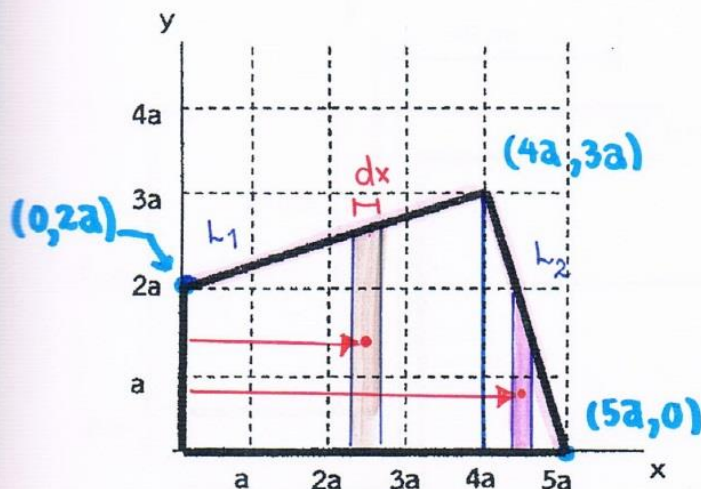


ชื่อ-สกุล.....รหัส.....ตอน.....

4. (20 คะแนน) จงหาดำแหน่งเซนทรอยด์ของพื้นที่ภายในเส้นทึบข้างล่างนี้ โดยวิธีการอินทิเกรต



$$\text{จาก } y - y_1 = m(x - x_1)$$

$$L_1: (y - 2a) = \frac{3a - 2a}{4a - 0} (x - 0)$$

$$y = \frac{1}{4}x + 2a$$

$$L_2: (y - 0) = \frac{3a - 0}{4a - 5a} (x - 5a)$$

$$y = -3x + 15a$$

$$\text{step 1 } \bar{x} = \frac{\int \tilde{x} dA}{\int dA}, \quad \bar{y} = \frac{\int \tilde{y} dA}{\int dA}$$

$$\text{step 2 } \bar{x} = \frac{\int \tilde{x}_1 dA_1 + \int \tilde{x}_2 dA_2}{\int dA_1 + \int dA_2}, \quad \bar{y} = \frac{\int \tilde{y}_1 dA_1 + \int \tilde{y}_2 dA_2}{\int dA_1 + \int dA_2}$$

$$\begin{aligned} \int dA_1 + \int dA_2 &= \int_0^{4a} \left[\frac{1}{4}x + 2a \right] dx + \int_{4a}^{5a} [-3x + 15a] dx \\ &= \left[\frac{1}{4} \frac{x^2}{2} + 2ax \right]_0^{4a} + \left[-\frac{3}{2} \frac{x^2}{2} + 15ax \right]_{4a}^{5a} \\ &= \left[\frac{1}{8} (16a^2) + 8a^2 \right] + \left[-\frac{3}{2} (5a)^2 + 15a(5a) \right] - \left[-\frac{3}{2} (4a)^2 + 15a(4a) \right] = 11.5a^2 \end{aligned}$$

$$\begin{aligned} \int \tilde{x}_1 dA_1 + \int \tilde{x}_2 dA_2 &= \int_0^{4a} x \left[\frac{1}{4}x + 2a \right] dx + \int_{4a}^{5a} x [-3x + 15a] dx \\ &= \left[\frac{1}{4} \frac{x^3}{3} + 2a \frac{x^2}{2} \right]_0^{4a} + \left[-\frac{3}{2} \frac{x^3}{3} + 15a \frac{x^2}{2} \right]_{4a}^{5a} \\ &= \left[\frac{1}{12} (4a)^3 + a(4a)^2 \right] + \left[-\frac{1}{2} (5a)^3 + \frac{15a}{2} (5a)^2 \right] - \left[-\frac{1}{2} (4a)^3 + \frac{15a}{2} (4a)^2 \right] \\ &= 27.8a^3 \end{aligned}$$