ไจทย์ เสริม Dynamics Final

Solution:

$$I_A = \frac{W_1}{g}\left[\frac{1}{4}\left(\frac{a}{2}\right)^2 + \frac{b^2}{12}\right] + \frac{W_1}{g}\left(l + \frac{b}{2}\right)^2 + \left(\frac{W_2}{g}\right)\frac{l^2}{3}$$

$$d = \frac{W_1\left(l + \frac{b}{2}\right) + W_2\left(\frac{l}{2}\right)}{W_1 + W_2}$$

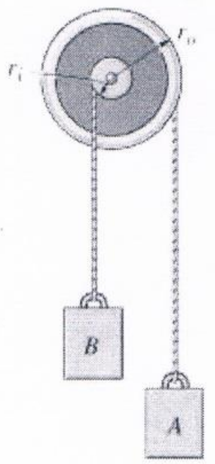
Guess $\theta_f = 1 \text{ deg}$

Given $\frac{1}{2}I_A\omega_0^2 - (W_1 + W_2)d\cos(\theta_0) = -(W_1 + W_2)d\cos(\theta_f)$

$\theta_f = \text{Find}(\theta_f)$ $\theta_f = 39.3 \text{ deg}$

Problem 18-47

The compound disk pulley consists of a hub and attached outer rim. If it has mass m_p and radius of gyration k_G, determine the speed of block A after A descends distance d from rest. Blocks A and B each have a mass m_b. Neglect the mass of the cords.



Given:

$m_p = 3 \text{ kg}$ $r_i = 30 \text{ mm}$ $m_b = 2 \text{ kg}$

$k_G = 45 \text{ mm}$ $r_o = 100 \text{ mm}$

$d = 0.2 \text{ m}$ $g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

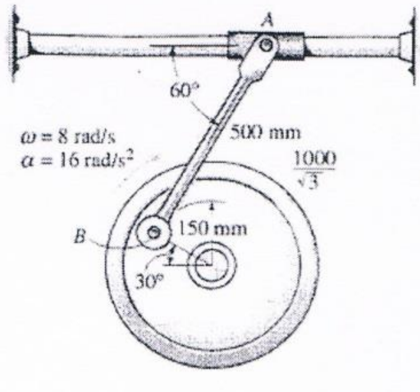
Guess $v_A = 1 \frac{\text{m}}{\text{s}}$

Given

$$0 + 0 = \frac{1}{2}m_b v_A^2 + \frac{1}{2}m_b\left(\frac{r_i}{r_o}v_A\right)^2 + \frac{1}{2}\left(m_p k_G^2\right)\left(\frac{v_A}{r_o}\right)^2 - m_b g d + m_b g\left(\frac{r_i}{r_o}\right)d$$

$v_A = \text{Find}(v_A)$ $v_A = 1.404 \frac{\text{m}}{\text{s}}$

R2-1. At a given instant, the wheel is rotating with the angular motions shown. Determine the acceleration of the collar at A at this instant.



$$\mathbf{a}_A = \mathbf{a}_B + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} + \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B})$$

$$v_B = 8(0.150) = 1.20 \text{ m/s}$$

$$\omega_{AB} = \frac{1.20}{\left(\frac{0.500}{\sqrt{3}}\right)} = 4.1569 \text{ rad/s}$$

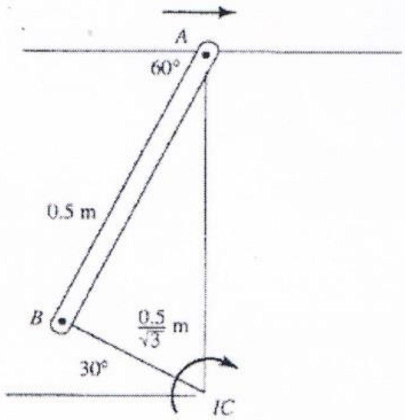
$$\underset{\rightarrow}{a_A} = \underset{\angle 60^\circ}{16(0.150)} + \underset{30^\circ}{(8)^2(0.150)} + \underset{30^\circ}{\alpha_{AB}(0.500)} + \underset{60^\circ}{(4.1569)^2(0.500)}$$

$$0 = 1.2\sqrt{3} - 4.8 - 0.250\alpha_{AB} - 4.32\sqrt{3}$$

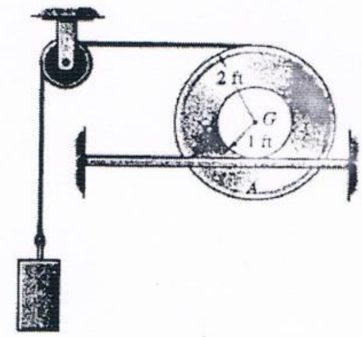
$$a_A = 1.2 + 4.8\sqrt{3} + (0.250\sqrt{3})\alpha_{AB} - 4.32$$

$$\alpha_{AB} = -40.8 \text{ rad/s}^2$$

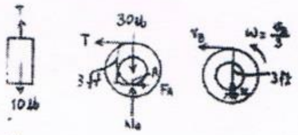
$$a_A = -12.5 \text{ m/s}^2 = 12.5 \text{ m/s}^2 \leftarrow$$ **Ans**



19-23. The inner hub of the wheel rests on the horizontal track. If it does not slip at A, determine the speed of the 10-lb block in 2 s after the block is released from rest. The wheel has a weight of 30 lb and a radius of gyration $k_G = 1.30$ ft. Neglect the mass of the pulley and cord.



Spool,



$(\circlearrowleft +) \quad (H_A)_1 + \Sigma\int M_A\,dt = (H_A)_2$

$$0 + T(3)(2) = [\frac{30}{32.2}(1.3)^2 + \frac{30}{32.2}(1)^2](\frac{v_B}{3})$$

Block,

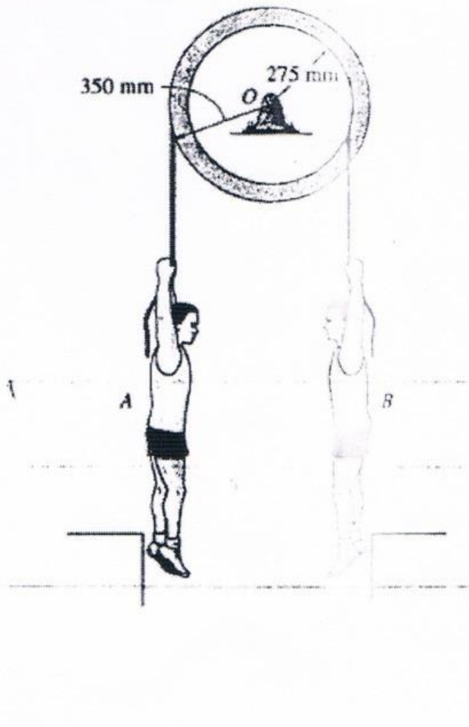
$(+\downarrow) \quad m(v_y)_1 + \Sigma\int F_y\,dt = m(v_y)_2$

$$0 + 10(2) - T(2) = \frac{10}{32.2}v_B$$

$v_B = 34.0$ ft/s $\qquad$ Ans

$T = 4.73$ lb

*19-32. The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass of 30 kg and a radius of gyration $k_O = 250$ mm. If two men A and B grab the suspended ropes and step off the ledges at the same time, determine their speeds in 4 s starting from rest. The men A and B have a mass of 60 kg and 70 kg, respectively. Assume they do not move relative to the rope during the motion. Neglect the mass of the rope.



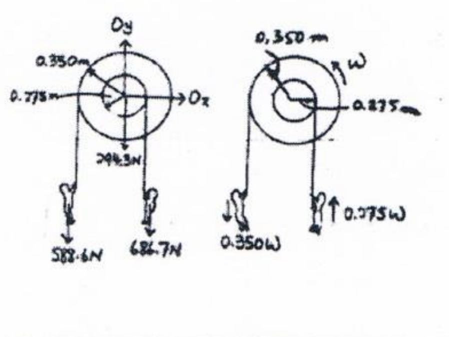
$(\circlearrowleft + \quad (H_O)_1 + \Sigma\int M_O\,dt = (H_O)_2$

$$0 + 588.6(0.350)(4) - 686.7(0.275)(4) = 30(0.25)^2\omega + 60(0.35\omega)(0.35) + 70(0.275\omega)(0.275$$

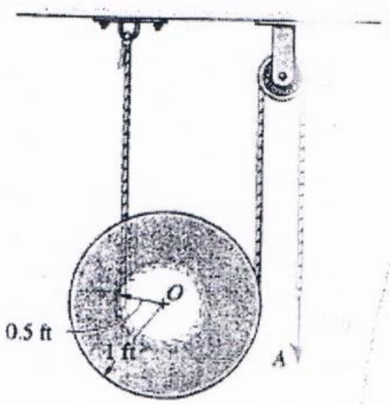
$\omega = 4.73$ rad/s

$v_A = 0.35(4.73) = 1.66$ m/s $\qquad$ Ans

$v_B = 0.275(4.73) = 1.30$ m/s $\qquad$ Ans



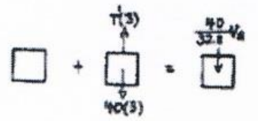
R2-30. Solve Prob. R2–29 if a 40-lb block is suspended from the cord at A, rather than applying the 40-lb force.



Block:

$(+\downarrow) \quad mv_1 + \Sigma \int F\,dt = mv_2$

$0 + 40(3) - T'(3) = \frac{40}{32.2} v_A \qquad (1)$



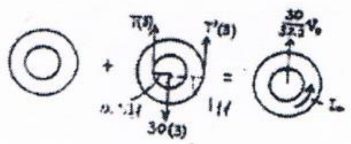
Spool:

$(+\uparrow) \quad mv_1 + \Sigma \int F\,dt = mv_2$

$0 + T(3) - 30(3) + T'(3) = \frac{30}{32.2} v_O$

$(\circlearrowleft +) \quad (H_O)_1 + \Sigma \int M_O\,dt = (H_O)_2$

$0 - T(0.5)3 + T'(1)3 = [\frac{30}{32.2}(0.65)^2]\omega$



Kinematics

$v_O = 0.5\omega$

$v_A = 1.5\omega$

Solving,

$T' = 15.5$ lb

$T = 20.6$ lb

$\omega = 39.5$ rad/s **Ans**

$v_O = 19.7$ ft/s

$v_A = 59.2$ ft/s

Also,

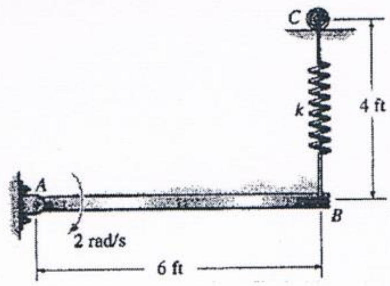
$(\circlearrowleft +) \; (H_{IC})_1 + \Sigma \int M_{IC}\,dt = (H_{IC})_2$

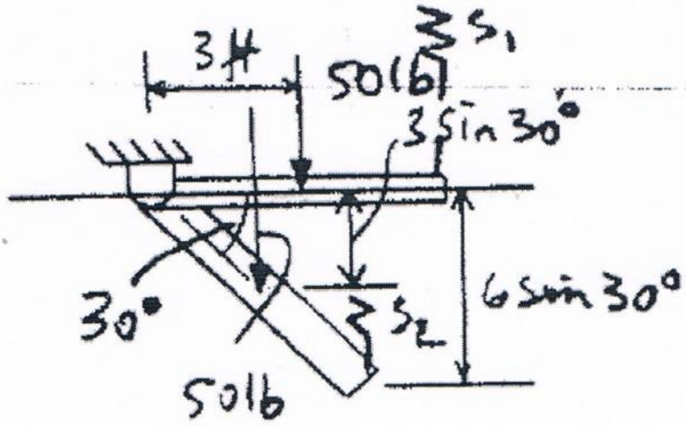
$0 + T'(1.5)(3) - 30(0.5)(3) = \left(\frac{30}{32.2}(0.65)^2 + \frac{30}{32.2}(0.5)^2\right)\omega$

Since $v_A = 1.5\omega$, then with Eq. (1) we get

$\omega = 39.5$ rad/s **Ans**

18-58. At the instant shown, the 50-lb bar is rotating downwards at 2 rad/s. The spring attached to its end always remains vertical due to the roller guide at C. If the spring has an unstretched length of 2 ft and a stiffness of $k = 12$ lb/ft, determine the angle θ, measured below the horizontal, to which the bar rotates before it momentarily stops.





$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{3}\left(\frac{50}{32.2}\right)(6)^2\right](2)^2 + \frac{1}{2}(12)(4-2)^2 = 0 + \frac{1}{2}(12)(4+6\sin\theta-2)^2 - 50(3\sin\theta)$$

$$61.2671 = 24(1+3\sin\theta)^2 - 150\sin\theta$$

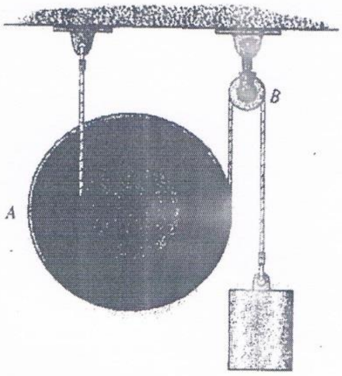
$$37.2671 = -6\sin\theta + 216\sin^2\theta$$

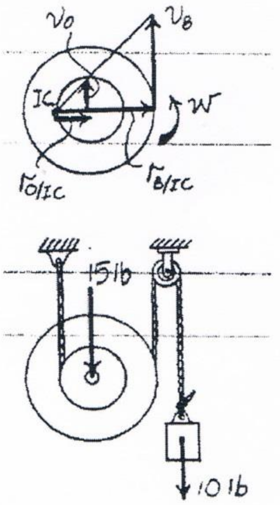
Set $x = \sin\theta$, and solve the quadratic equation for the positive root:

$$\sin\theta = 0.4295$$

$\theta = 25.4°$ **Ans**

***18-32.** Pulley A weighs 15 lb and has a radius of gyration of $k_O = 0.8$ ft. If the system is released from rest, determine the velocity of the center O of the pulley after the 10-lb block moves downward 4 ft. Neglect the mass of the pulley at B.





$$\frac{s_O}{0.5} = \frac{4}{1.5} \qquad s_O = 1.333 \text{ ft}$$

$$\omega = \frac{v_O}{r_{O/IC}} = \frac{v_B}{r_{B/IC}}$$

$$\omega = \frac{v_O}{0.5} = 2v_O \text{ and } \frac{v_O}{0.5} = \frac{v_B}{1.5}, \; v_B = 3v_O.$$

Principle of Work and Energy : The mass moment of inertia of the pulley about point O is $I_O = mk_O^2 = \left(\frac{15}{32.2}\right)(0.8^2) = 0.2981 \text{ slug}\cdot\text{ft}^2$. Applying Eq. 18 − 13, we have

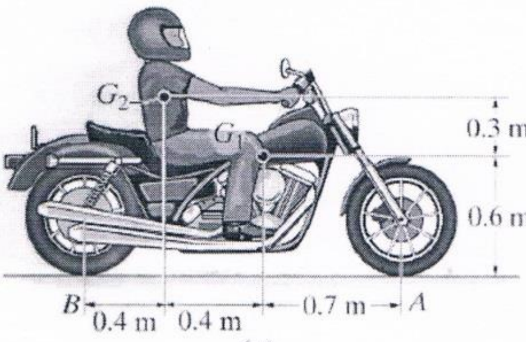
$$T_1 + \sum U_{1-2} = T_2$$

$$0 + W_B s_B - W_P s_O = \frac{1}{2} m_B v_B^2 + \frac{1}{2} m_P v_O^2 + \frac{1}{2} I_O \omega^2$$

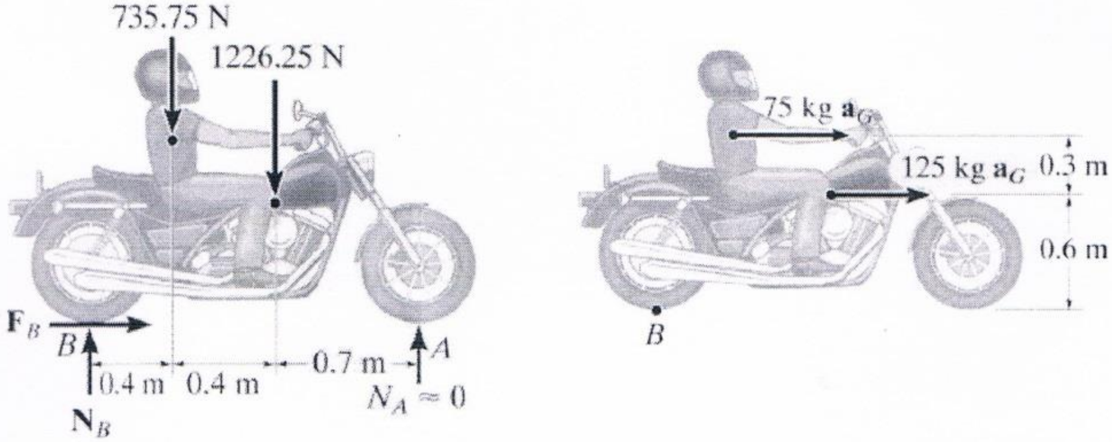
$$0 + 10(4) - 15(1.333) = \frac{1}{2}\left(\frac{10}{32.2}\right)(3v_O)^2 + \frac{1}{2}\left(\frac{15}{32.2}\right)v_O^2 + \frac{1}{2}(0.2981)(2v_O)^2$$

$$v_O = 3.00 \text{ ft/s} \qquad \textbf{Ans}$$

The motorcycle shown in Fig. 17–11*a* has a mass of 125 kg and a center of mass at G_1, while the rider has a mass of 75 kg and a center of mass at G_2. Determine the minimum coefficient of static friction between the wheels and the pavement in order for the rider to do a "wheely," i.e., lift the front wheel off the ground as shown in the photo. What acceleration is necessary to do this? Neglect the mass of the wheels and assume that the front wheel is free to roll.



(a)



Equations of Motion.

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad F_B = (75\text{ kg} + 125\text{ kg})a_G \qquad (1)$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_B - 735.75\text{ N} - 1226.25\text{ N} = 0$

$\circlearrowleft + \Sigma M_B = \Sigma(\mathcal{M}_k)_B;\ -(735.75\text{ N})(0.4\text{ m}) - (1226.25\text{ N})(0.8\text{ m}) =$

$-(75\text{ kg } a_G)(0.9\text{ m}) - (125\text{ kg } a_G)(0.6\text{ m}) \qquad (2)$

Solving,

$$a_G = 8.95\text{ m/s}^2 \rightarrow \qquad \textit{Ans.}$$

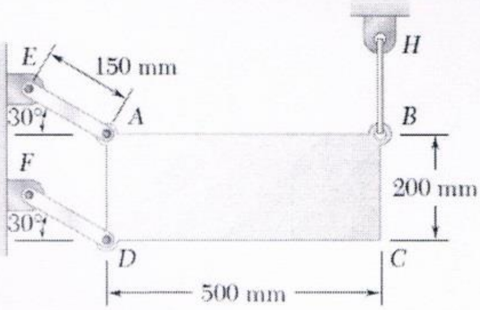
$$N_B = 1962\text{ N}$$

$$F_B = 1790\text{ N}$$

Thus the minimum coefficient of static friction is

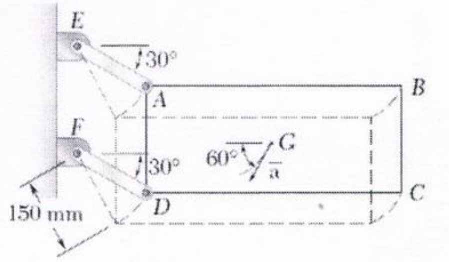
$$(\mu_s)_{\min} = \frac{F_B}{N_B} = \frac{1790\text{ N}}{1962\text{ N}} = 0.912 \qquad \textit{Ans.}$$

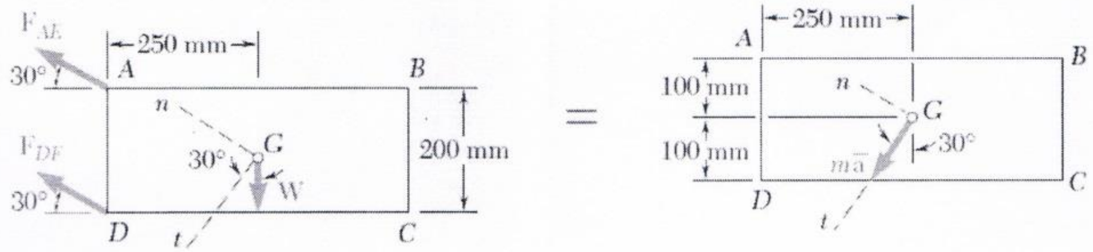
SAMPLE PROBLEM 16.2



The thin plate $ABCD$ of mass 8 kg is held in the position shown by the wire BH and two links AE and DF. Neglecting the mass of the links, determine immediately after wire BH has been cut (a) the acceleration of the plate, (b) the force in each link.

SOLUTION





a. Acceleration of the Plate.

$+\swarrow \Sigma F_t = \Sigma (F_t)_{\text{eff}}$:

$$W \cos 30° = m\bar{a}$$

$$mg \cos 30° = m\bar{a}$$

$$\bar{a} = g \cos 30° = (9.81 \text{ m/s}^2) \cos 30° \qquad (1)$$

$$\bar{a} = 8.50 \text{ m/s}^2 \nearrow 60° \quad \blacktriangleleft$$

b. Forces in Links *AE* and *DF*.

$+\nwarrow \Sigma F_n = \Sigma (F_n)_{\text{eff}}$: $\quad F_{AE} + F_{DF} - W \sin 30° = 0 \qquad (2)$

$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}$:

$$(F_{AE} \sin 30°)(250 \text{ mm}) - (F_{AE} \cos 30°)(100 \text{ mm}) + (F_{DF} \sin 30°)(250 \text{ mm}) + (F_{DF} \cos 30°)(100 \text{ mm}) = 0$$

$$38.4 F_{AE} + 211.6 F_{DF} = 0$$

$$F_{DF} = -0.1815 F_{AE} \qquad (3)$$

Substituting for F_{DF} from (3) into (2), we write

$$F_{AE} - 0.1815 F_{AE} - W \sin 30° = 0$$

$$F_{AE} = 0.6109W$$

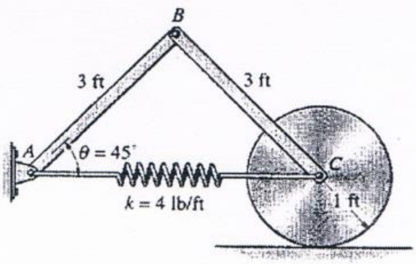
$$F_{DF} = -0.1815(0.6109W) = -0.1109W$$

Noting that $W = mg = (8 \text{ kg})(9.81 \text{ m/s}^2) = 78.48$ N, we have

$$F_{AE} = 0.6109(78.48 \text{ N}) \qquad F_{AE} = 47.9 \text{ N } T \quad \blacktriangleleft$$

$$F_{DF} = -0.1109(78.48 \text{ N}) \qquad F_{DF} = 8.70 \text{ N } C \quad \blacktriangleleft$$

18-30. The assembly consists of two 15-lb slender rods and a 20-lb disk. If the spring is unstretched when $\theta = 45°$ and the assembly is released from rest at this position, determine the angular velocity of rod AB at the instant $\theta = 0°$. The disk rolls without slipping.

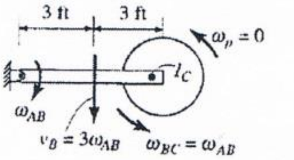


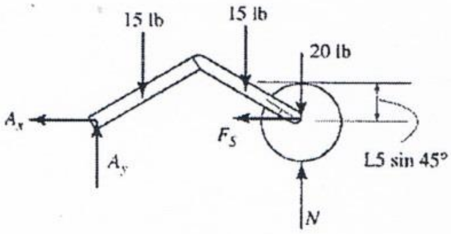
$$T_1 + \sum U_{1-2} = T_2$$

$$[0+0] + 2(15)(1.5)\sin 45° - \frac{1}{2}(4)[6 - 2(3)\cos 45°]^2$$

$$= 2\left[\frac{1}{2}\left(\frac{1}{3}\left(\frac{15}{32.2}\right)(3)^2\right)\omega_{AB}^2\right]$$

$\omega_{AB} = 4.28$ rad/s **Ans**

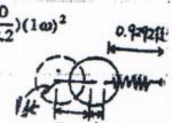




18-43. The 50-lb wheel has a radius of gyration about its center of gravity G of $k_G = 0.7$ ft. If it rolls without slipping, determine its angular velocity when it has rotated clockwise 90° from the position shown. The spring AB has a stiffness $k = 1.20$ lb/ft and an unstretched length of 0.5 ft. The wheel is released from rest.

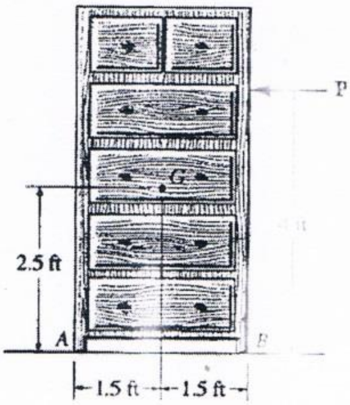
$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(1.20)[\sqrt{(3)^2 + (0.5)^2} - 0.5]^2 = \frac{1}{2}[\frac{50}{32.2}(0.7)^2]\omega^2 + \frac{1}{2}(\frac{50}{32.2})(1\omega)^2 + \frac{1}{2}(1.20)(0.9292 - 0.5)^2$$



$\omega = 1.80$ rad/s **Ans**

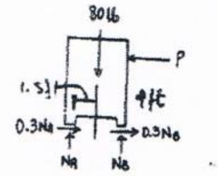
R2-31. The dresser has a weight of 80 lb and is pushed along the floor. If the coefficient of static friction at A and B is $\mu_s = 0.3$ and the coefficient of kinetic friction is $\mu_k = 0.2$, determine the smallest horizontal force P needed to cause motion. If this force is increased slightly, determine the acceleration of the dresser. Also, what are the normal reactions at A and B when it begins to move?



For slipping:

$\xrightarrow{+} \Sigma F_x = 0; \quad -P + 0.3(N_A + N_B) = 0$

$+\uparrow \Sigma F_y = 0; \quad N_A + N_B - 80 = 0$



$P = 24$ lb **Ans**

For tipping $N_B = 0, \quad N_A = 80$ lb

$\circlearrowleft + \Sigma M_A = 0; \quad P(4) - 80(1.5) = 0$

$P = 30 \text{ lb} < 24 \text{ lb}$

Therefore dresser slips.

$+\uparrow \Sigma F_y = 0; \quad N_A + N_B - 80 = 0$

$\xleftarrow{+} \Sigma F_x = ma_x; \quad 24 - 0.2N_A - 0.2N_B = \frac{80}{32.2}a_G$

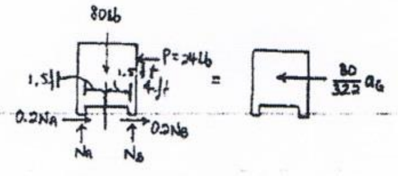
$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad 24(4) + N_B(3) - 80(1.5) = \frac{80}{32.2}a_G(2.5)$

Solving,

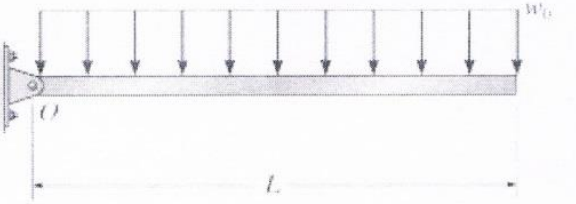
$a_G = 3.22 \text{ ft/s}^2$ **Ans**

$N_B = 14.8$ lb **Ans**

$N_A = 65.3$ lb **Ans**



Solution:



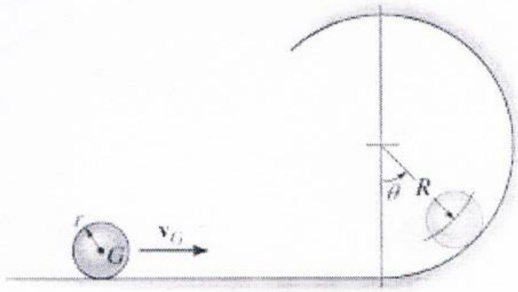
$$\int_0^{\frac{\pi}{2}}\int_0^L x w_0 \,dx\, d\theta = \frac{1}{4}\pi L^2 w_0$$

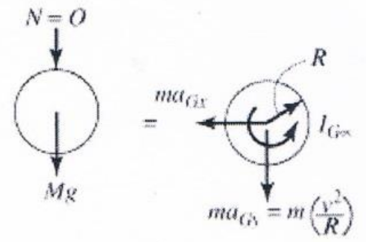
(a) $\frac{1}{4}\pi L^2 w_0 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$ $\qquad\omega = \sqrt{\frac{3\pi w_0}{2m}}$

(b) $\frac{1}{4}\pi L^2 w_0 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 - mg\frac{L}{2}$ $\qquad\omega = \sqrt{\frac{3\pi w_0}{2m} + \frac{3g}{L}}$

Problem 18-33

A ball of mass m and radius r is cast onto the horizontal surface such that it rolls without slipping. Determine the minimum speed v_G of its mass center G so that it rolls completely around the loop of radius $R + r$ without leaving the track.





Solution:

$$mg = m\left(\frac{v^2}{R}\right) \qquad v^2 = gR$$

$$\frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_G}{r}\right)^2 + \frac{1}{2}mv_G^2 - mg2R = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2$$

$$\frac{1}{5}v_G^2 + \frac{1}{2}v_G^2 = 2gR + \frac{1}{5}gR + \frac{1}{2}gR \qquad v_G = 3\sqrt{\frac{3}{7}gR}$$

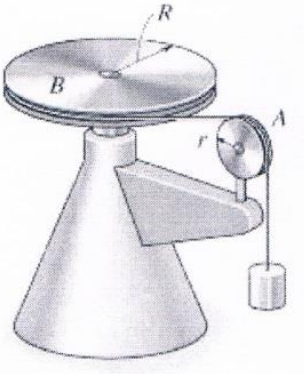
Given $$W\left(\frac{\sqrt{a^2+b^2}}{2}\right) = \frac{1}{2}\frac{W}{g}\left(\frac{a^2+b^2}{3}\right)\omega^2 + W\frac{a}{2}$$ $\omega = \text{Find}(\omega)$

$v_A = \omega b$ $\qquad v_A = 11.9\ \frac{\text{ft}}{\text{s}}$

Problem 18-50

The assembly consists of pulley A of mass m_A and pulley B of mass m_B. If a block of mass m_b is suspended from the cord, determine the block's speed after it descends a distance d starting from rest. Neglect the mass of the cord and treat the pulleys as thin disks. No slipping occurs.

Given:



$m_A = 3\ \text{kg}$

$m_B = 10\ \text{kg}$

$m_b = 2\ \text{kg}$

$d = 0.5\ \text{m}$

$r = 30\ \text{mm}$

$R = 100\ \text{mm}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution: Guess $v_b = 1\ \frac{\text{m}}{\text{s}}$

Given

$$0 + 0 = \frac{1}{2}\left(\frac{m_A r^2}{2}\right)\left(\frac{v_b}{r}\right)^2 + \frac{1}{2}\left(\frac{m_B R^2}{2}\right)\left(\frac{v_b}{R}\right)^2 + \frac{1}{2}m_b v_b^2 - m_b g d$$

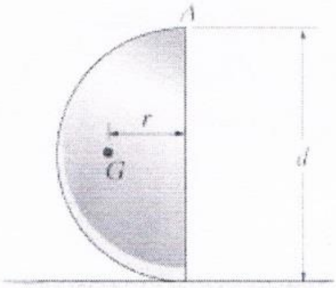
$v_b = \text{Find}(v_b)$ $\qquad v_b = 1.519\ \frac{\text{m}}{\text{s}}$

Problem 18-51

A uniform ladder having weight W is released from rest when it is in the vertical position. If it is allowed to fall freely, determine the angle at which the bottom end A starts to lift off the ground. For the calculation, assume the ladder to be a slender rod and neglect friction at A.

***Problem 18-48**

The semicircular segment of mass M is released from rest in the position shown. Determine the velocity of point A when it has rotated counterclockwise 90°. Assume that the segment rolls without slipping on the surface. The moment of inertia about its mass center is I_G.



Given:

$M = 15\ \text{kg}$ $\quad r = 0.15\ \text{m}$

$I_G = 0.25\ \text{kg}\cdot\text{m}^2$ $\quad d = 0.4\ \text{m}$

Solution:

Guesses $\quad \omega = 1\ \frac{\text{rad}}{\text{s}} \quad v_G = 1\ \frac{\text{m}}{\text{s}}$

Given $\quad Mgd = \frac{1}{2}Mv_G^2 + \frac{1}{2}I_G\omega^2 + Mg(d-r) \quad v_G = \omega\left(\frac{d}{2} - r\right)$

$$\begin{pmatrix}\omega \\ v_G\end{pmatrix} = \text{Find}(\omega, v_G) \qquad \omega = 12.4\ \frac{\text{rad}}{\text{s}} \qquad v_G = 0.62\ \frac{\text{m}}{\text{s}}$$

$$\mathbf{v_A} = \begin{pmatrix}0 \\ 0 \\ \omega\end{pmatrix} \times \begin{pmatrix}\frac{-d}{2} \\ \frac{d}{2} \\ 0\end{pmatrix} \qquad \mathbf{v_A} = \begin{pmatrix}-2.48 \\ -2.48 \\ 0.00\end{pmatrix}\frac{\text{m}}{\text{s}} \qquad |\mathbf{v_A}| = 3.50\ \frac{\text{m}}{\text{s}}$$

Problem 18-49

The uniform stone (rectangular block) of weight W is being turned over on its side by pulling the vertical cable *slowly* upward until the stone begins to tip. If it then falls freely (**T** = 0) from an essentially balanced at-rest position, determine the speed at which the corner A strikes the pad at B. The stone does not slip at its corner C as it falls.

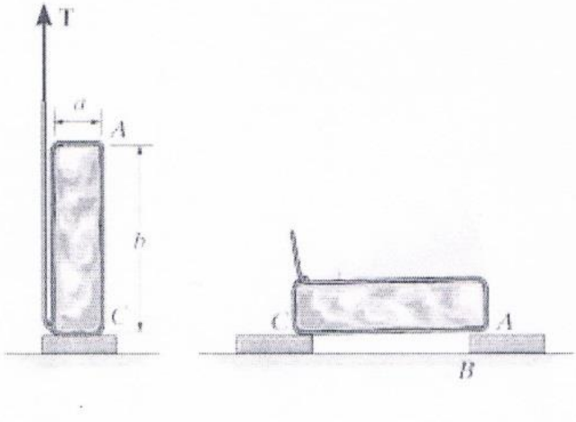
Given:

$W = 150\ \text{lb}$

$a = 0.5\ \text{ft}$

$b = 2\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

Guess $\quad \omega = 1\ \frac{\text{rad}}{\text{s}}$