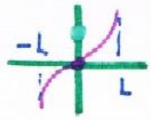
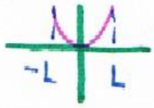
Reviews (Fourier)

(1) Odd f^n : $f(-x) = -f(x) \rightarrow \sin x$,  $\Rightarrow \int_{-L}^{L} f(x)\,dx = 0$

Even f^n : $f(-x) = f(x) \rightarrow \cos x$,  $\Rightarrow \int_{-L}^{L} f(x)\,dx = 2\int_{0}^{L} f(x)\,dx$

(2) $(-\pi, \pi)$ → $\langle f, g \rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)g(x)\,dx$

→ $\|f\| = \sqrt{\langle f, f \rangle}$

(3) Orthogonal : $\langle f, g \rangle = 0$

Orthogonal set : $\langle f_n, f_m \rangle = 0$ ระหว่าง f

$\{ f_n \mid n = 1, 2, \ldots \}$

Orthonomal set : $\|f_n\| = 1$ สมาชิกทุกตัว

(4) Fourier series : $T = 2L$

$\boxed{(-\pi, \pi)}$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$

$\boxed{(-L, L)}$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L}\right)$$

$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$

$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos \frac{n\pi x}{L}\,dx$

$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin \frac{n\pi x}{L}\,dx$

Ex $(-2, 2) \rightarrow L = 2$

$(0, 6) \rightarrow L = 3$

(5) Trick 1

1) $f(x) \sim \Sigma(\sin)$ F. sin → odd

2) $f(x) \sim \Sigma(\cos)$ F. cos → even

3) $f(x) \sim \Sigma(\cos \ldots \sin)$ F → not odd, even

Trick 2 หาค่า coefficient of Fourier : $\int_{-L}^{L} f(x) \underset{\cos\ \ \sin}{\ldots\ldots}\,dx$

$\Rightarrow \boxed{La_0}$ or $\boxed{La_n}$ or $\boxed{Lb_n}$

Trick 3 การหา ผลบวก อนุกรมแปลกๆ

* ต้องการ cos → กำจัด sin

ต้องการ sin → กำจัด cos *

$f(x) \sim \Sigma(\cos \ldots \sin)$

- $x = x_0 \rightarrow f(x_0)$: x_0 ต่อเนื่อง
- $x = x_0 \rightarrow \frac{f(x_0^+) + f(x_0^-)}{2}$: x_0 ไม่ต่อเนื่อง → ตัวแทน $f(x_0)$

* (6) Fourier Extension : $f(x)$ defined on $\boxed{(0, L)}$ and need

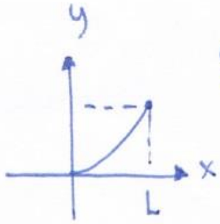
- odd Extended
- even Extended
- F.Sin
- F.Cos

} $F(x)$

- Fourier sine (odd extened) : $f(x) \sim \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$ → $b_n = \frac{2}{L}\int_0^L f(x) \sin \frac{n\pi x}{L}\, dx$
- Fourier cosine (even extened) : $f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$ → $a_0 = \frac{2}{L}\int_0^L f(x)\, dx$

$a_n = \frac{2}{L}\int_0^L f(x) \cos \frac{n\pi x}{L}\, dx$

Ex



$y = x^2, \quad 0 < x < L$

→ even extend  $F(x) \rightarrow$ even

→ odd extend  $F(x) \rightarrow$ odd