BRIGHT KIDS
Academic of science

PHYSICS-MATHS BY P'JUE

Dynamics

Engineering mechanics 2

PRE FLIGHT

KINEMATICS OF A PARTICLE

P'JUE

จ. Fcndc พ. Solid พฤ stat

PRE FLIGHT DYNAMICS-57 : Kinematics of Particles

Drill : Rectilinear..

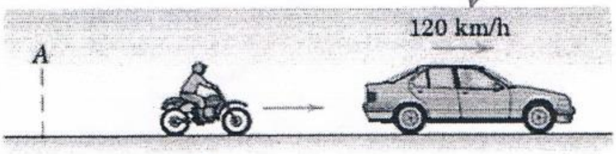
ตำรวจจราจร

2/32 A motorcycle patrolman starts from rest at A two seconds after a car, speeding at the constant rate of 120 km/h, passes point A. If the patrolman accelerates at the rate of 6 m/s² until he reaches his maximum permissible speed of 150 km/h, which he maintains, calculate the distance s from point A to the point at which he overtakes the car.

คงที่ต่อไป

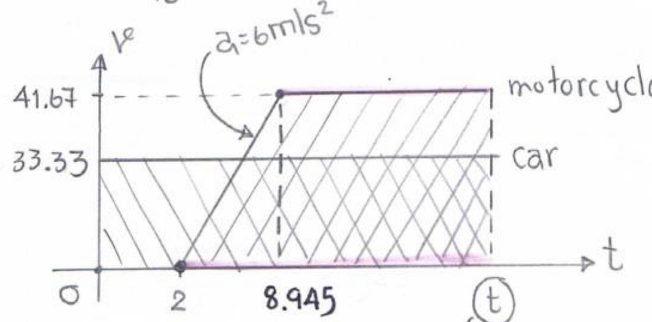
ทัน

คงที่



$120 \times \frac{5}{18} = 33.33$ m/s

$150 \times \frac{5}{18} = 41.67$ m/s



$a = 6$ m/s² ----> $v = 41.67$ m/s

$a = \frac{v-u}{t}$

$6 = \frac{41.67-0}{t}$

$t = 6.945$ s

ทันกัน $\quad S_{car} = S_{motor}$

$Area_{car} = Area_{motor}$

$33.33t = \frac{1}{2}[(t-2)+(t-8.94)]41.67$

$t = 27.23$ s

$\therefore \boxed{S_{car} = 33.33 \times 27.23 = 907.6 \text{ m}}$ #

12-21. A particle travels in a straight line with accelerated motion such that $a = -ks$, where s is the distance from the starting point and k is a proportionality constant which is to be determined. For $s = 2$ ft the velocity is 4 ft/s, and for $s = 3.5$ ft the velocity is 10 ft/s. What is s when $v = 0$?

$s = 2$ ft $\quad v = 4$ ft/s ✓

$s = 3.5$ ft $\quad v = 10$ ft/s

$s = ?$ $\quad v = 0$

$v\,dv = a\,ds$

$\int_4^v v\,dv = \int_2^s -ks\,ds$

$\frac{v^2}{2}\Big|_4^v = -k\frac{s^2}{2}\Big|_2^s$

$\frac{v^2}{2} - \frac{4^2}{2} = -\frac{k}{2}(s^2-2^2)$

$v^2 - 16 = -k(s^2-4)$ — ①

หา k

$10^2 - 16 = -k(3.5^2-4)$

$\boxed{k = -10.18 \ s^{-2}}$

หา s เมื่อ $v=0$

$0 - 16 = -(-10.18)(s^2-4)$

$\frac{-16}{10.18} = s^2-4$

$\boxed{s = 1.56 \text{ ft}}$

12-29. A particle is moving along a straight line such that when it is at the origin it has a velocity of 4 m/s. If it begins to decelerate at the rate of $a = (-1.5v^{1/2})\ \text{m/s}^2$, where v is in m/s, determine the distance it travels before it stops.

$v_0 = 4\ \text{m/s} \qquad v = 0 \qquad s = ?$

$a = -1.5\,v^{1/2}\ \text{m/s}^2$

$v\,dv = a\,ds$

$$\int_0^s ds = \int_4^v \frac{v}{-1.5\,v^{1/2}}\,dv$$

$$s - 0 = \int_4^v \frac{v^{1/2}}{-1.5}\,dv$$

$$s - 0 = \frac{1}{-1.5}\left[\frac{2}{3}v^{3/2}\right]_4^v$$

$$s = -\frac{2}{4.5}\left[v^{3/2} - 4^{3/2}\right] \quad \text{แทน } v = 0$$

$$\therefore\ s = -\frac{2}{4.5}\left[0 - 2^3\right] = \boxed{3.56\ \text{m}}$$ #

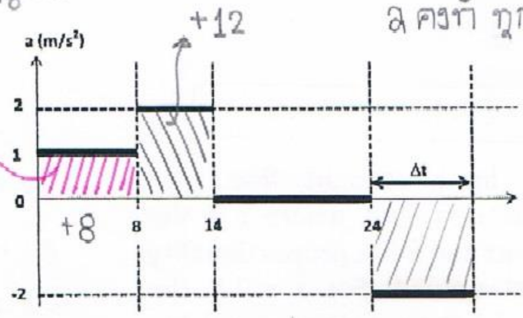
1.2 รถยนต์เคลื่อนที่จากหยุดนิ่งด้วยความเร่งดังในกราฟ จงหาเวลา Δt ที่รถยนต์เบรคจนหยุดนิ่ง และระยะทางที่เคลื่อนที่ทั้งหมด

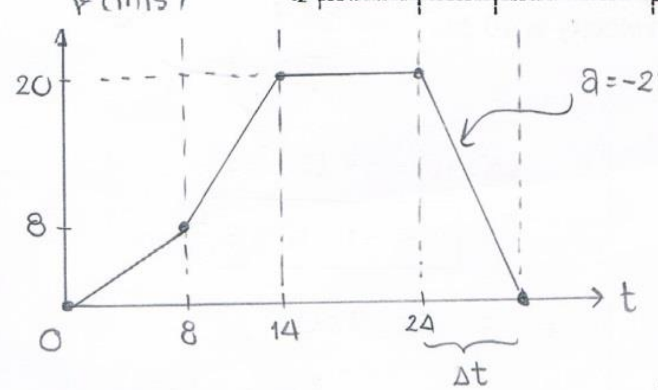
$v_0 = 0$

a คงที่ ทุกช่วง → 5 สูตร

พ.ท. = Δv

$\int a\,dt = v - v_0$





$a = -2\ \text{m/s}^2$

$$a = \frac{v - v_0}{t}$$

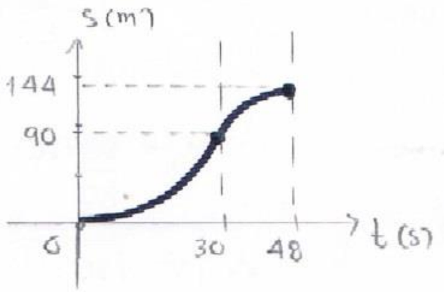
$$-2 = \frac{0 - 20}{\Delta t}$$

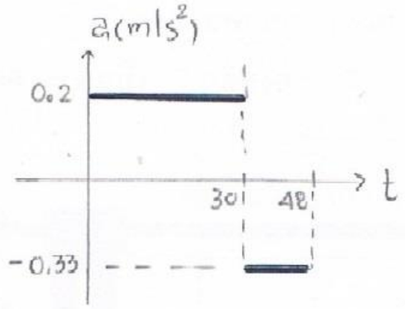
$$\boxed{\Delta t = 10\ \text{s}}$$ #

หา S : $s = \text{Area}$

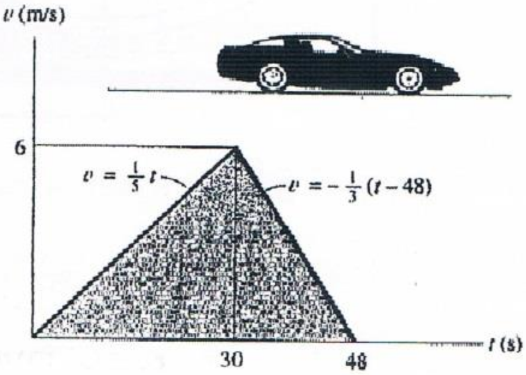
$$= \frac{1}{2}(8)8 + \frac{1}{2}(8+20)6 + 10(20) + \frac{1}{2}(10)20$$

$$\boxed{s = 386\ \text{m}}$$ #





12-51. A car travels along a straight road with the speed shown by the v–t graph. Determine the total distance the car travels until it stops when $t = 48$ s. Also plot the s–t and a–t graphs.



$0 \le t < 30\,s$

$v = \frac{1}{5}t$

$\int_0^s ds = \int_0^t v\,dt$

$s = \frac{1}{5}\frac{t^2}{2}\Big|_0^t$

$\boxed{s = 0.1t^2}$

$s(30s) = 90\,m$

$a = \frac{dv}{dt}$

$\boxed{a = \frac{1}{5} = 0.2\,\frac{m}{s^2}}$

$30 \le t \le 48\,s$

$v = -\frac{1}{3}(t-48)$

$\int_{90}^s ds = \int_{30}^t \left(-\frac{1}{3}t + 16\right) dt$

$s - 90 = \left(-\frac{1}{3}\frac{t^2}{2} + 16t\right)\Big|_{30}^t$

$s - 90 = \left(-\frac{1}{6}t^2 + 16t\right) - \left(-\frac{1}{6}\times 900 + 16(30)\right)$

$\boxed{s = -\frac{1}{6}t^2 + 16t - 240}$

$s(48s) = 144\,m$

$\boxed{a = -\frac{1}{3}\,m/s^2}$

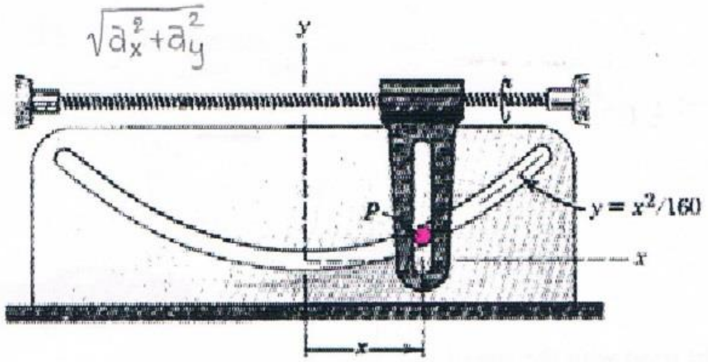
$$\frac{dx^2}{dt} = 2x\frac{dx}{dt}$$

Drill : Curvilinear……………………………………………

2.18 ในการเคลื่อนที่หนึ่ง สลัก P ถูกผลักให้เคลื่อนที่ในร่องพาราโบลา โดยแขนในแนวดิ่ง ซึ่งเคลื่อนที่ในทิศ x ด้วยความเร็วคงที่ 20 mm/s โดยหน่วยวัดทั้งหมด คือ มิลลิเมตร และวินาที จงคำนวณหา ขนาดของความเร็ว $\vec{v}$ และความเร่ง $\vec{a}$ ของสลัก P เมื่อ x = 60 mm

$\sqrt{v_x^2+v_y^2}$

$\sqrt{a_x^2+a_y^2}$



จุด P มีสมการการ ค.ท. เป็น $y = \frac{x^2}{160}$

v_x คงที่ $= 20$ mm/s

↳ $a_x = 0$ mm/s²

$$y = \frac{x^2}{160} \quad —(1)$$

$$\frac{d(1)}{dt};\quad v_y = \frac{2x\,v_x}{160} = \frac{x\,v_x}{80} \quad —(2)$$

$$\frac{d(2)}{dt};\quad a_y = \frac{1}{80}\left[x\,a_x + v_x\cdot v_x\right]$$

$$v_y = \frac{60(20)}{80} = \frac{120}{8} = 15\ \text{mm/s}$$

$$\therefore \boxed{v = \sqrt{20^2+15^2} = 25\ \text{mm/s}}$$

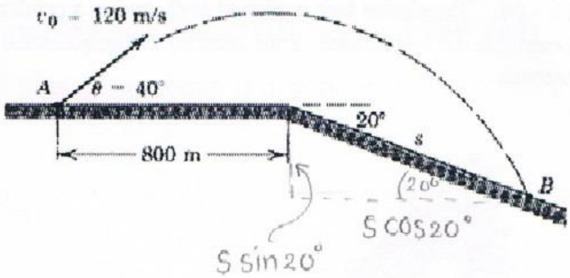
$$a_y = \frac{1}{80}\left[60(0) + 20^2\right]$$

$$a_y = 5\ \text{mm/s}^2$$

$$\therefore \boxed{a = \sqrt{0^2+5^2} = 5\ \text{mm/s}^2}$$

1. A particle travels along a path (เส้นทาง) described (อธิบาย) by the parabola $y = 0.5x^2$. The x-component of velocity is given by $v_x = 5t$ m/s. When t = 0, x = y = 0. Determine (จงหา) the particle's distance (ระยะทาง) from the origin (จุดต้น) and the magnitude (ขนาด) of its acceleration (ความเร่ง) when t=1s.

2.17 การเคลื่อนที่แบบโปรเจ็กไตล์เริ่มต้นจากจุด A ด้วยความเร็ว $v_0 = 120$ m/s และ $\theta = 40^\circ$ ดังแสดงในรูป
จงหาระยะทางตามทางลาดเอียง s ที่แสดงตำแหน่งกระทบที่ B และคำนวณหาเวลา t ของการเคลื่อนที่



x: $S_x = u_x t$

$(800 + S\cos 20^\circ) = 120\cos 40^\circ\, t$ — ①

y: $S_y = u_y t + \frac{1}{2} g t^2$

$-S\sin 20^\circ = (120\sin 40^\circ)t + \frac{1}{2}(-g)t^2$ — ②

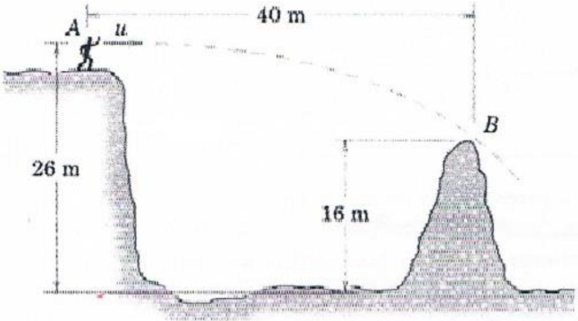
จาก ① : $S = 97.82t - 851.3$ แทนใน ②

$-(97.82t - 851.3)\sin 20^\circ = 77.13t - 4.91t^2$

$4.91t^2 - 110.59t + 291.2 = 0 \rightarrow t = 19.5, \cancel{3.04}$ s

$\boxed{t = 19.5 \text{ s},\ S = 1056 \text{ m}}$

2/72 With what minimum horizontal velocity u can a boy throw a rock at A and have it just clear the obstruction at B?



x: $S_x = U_x t$

$40 = ut$ — ① $+\downarrow$

y: $S_y = U_y t + \frac{1}{2} g t^2$

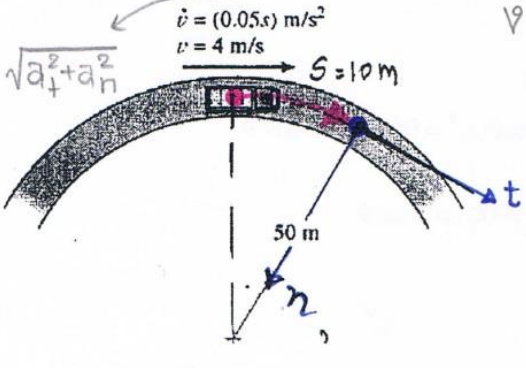
$+10 = 0 + \frac{1}{2}(+g)t^2$ — ②

$t = 1.43$ s แทนใน ①

$\therefore\ 40 = V_0(1.43) \rightarrow \boxed{V_0 = 28.01 \text{ m/s}}$

17. The truck (รถบรรทุก) travel at a speed of 4 m/s (ที่ความเร็ว 4 m/s) along a circular road (ถนนวงกลม) that has a radius of 50 m. For a short distance from s = 0 m, (สำหรับระยะทางสั้น ๆจาก 0 เมตร) its speed is then increased by (ความเร็วของรถมีค่าเพิ่มขึ้น) $\dot{v} = 0.05s$ m/s². Determine its speed (จงหาอัตราเร็วของรถ) and magnitude (ขนาด) of its acceleration (ความเร่ง) when it has moved s = 10 m. (เมื่อรถเคลื่อนที่ไปได้ 10 เมตร) $\rightarrow v_t$

$a = \sqrt{a_t^2 + a_n^2}$



$\dot{v} = \frac{dv}{dt} = \underbrace{a = 0.05s}_{a_t}$ m/s²

$v\,dv = a\,ds$

$\int_4^v v\,dv = \int_0^s 0.05s\,ds$

$\frac{v^2}{2}\Big|_4^v = 0.05\frac{s^2}{2}\Big|_0^s$

$\frac{v^2}{2} - \frac{4^2}{2} = \frac{0.05s^2}{2}$

$a_t = 0.05(10) = 0.5$ m/s²

$a_n = \frac{v^2}{\rho} = \frac{4.58^2}{50} = 0.42$ m/s²

$\therefore\ a = \sqrt{0.5^2 + 0.42^2}$

$\boxed{a = 0.653 \text{ m/s}^2}$

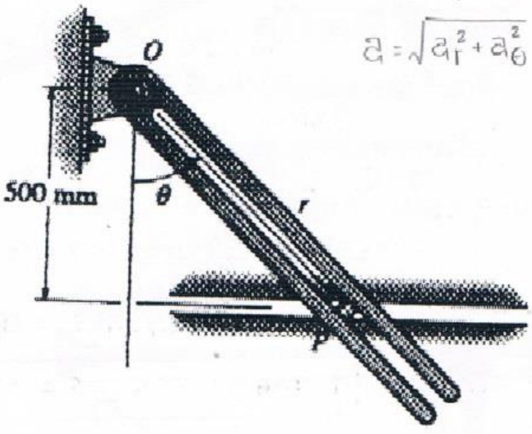
$S = 10$ m, $\frac{v^2}{2} - 8 = \frac{0.05(10)^2}{2}$

$\boxed{v_t = 4.58 \text{ m/s}}$

12 - 24. The slotted link is pinned at O, and as a result of rotation it drives the peg P along the horizontal guide. Determine the magnitudes of the velocity and acceleration of P as a function of θ if $\theta = (3t)$ rad, where t is in seconds.

$v = \sqrt{v_r^2 + v_\theta^2}$

$a = \sqrt{a_r^2 + a_\theta^2}$



ต้องหาสมการ เอง $< \begin{matrix} r(t) \\ r(\theta) \end{matrix}$ แล้วแต่

$r \quad \theta = 3t$

$\dot{r} \quad \dot{\theta}$

$\ddot{r} \quad \ddot{\theta}$

Here $r = (500/\cos\,\theta)$mm $= (500\sec\,\theta)$mm.

In the following use the identity $1 + \tan^2\theta = \sec^2\theta$ and recall $d(\sec\theta) = \sec\theta\tan\theta\,d\theta$, and $d(\tan\theta) = \sec^2\theta\,d\theta$.

$\dot{\theta} =$ 3

$\ddot{\theta} =$ 0

$\dot{r} =$ $500\sec\theta \cdot \tan\theta \cdot \dot{\theta}$ $= 1500\sec\theta\tan\theta$

$\ddot{r} =$ $1500[\sec\theta \cdot \sec^2\theta \cdot \dot{\theta} + \tan\theta \cdot \sec\theta \tan\theta \cdot \dot{\theta}]$ $= 4500(\sec\theta)(2\tan^2\theta + 1)$

$\hookrightarrow 1500(\sec\theta)(3)[\sec^2\theta + \tan^2\theta]$

$v_r = \dot{r} = 1500\sec\theta\tan\theta$

$v_\theta = r\dot{\theta} = 1500\sec\theta$

$v = \sqrt{(1500)^2\sec^2\theta\tan^2\theta + (1500)^2\sec^2\theta}$

$v = 1500\sec^2\theta$ mm/s **Ans.**

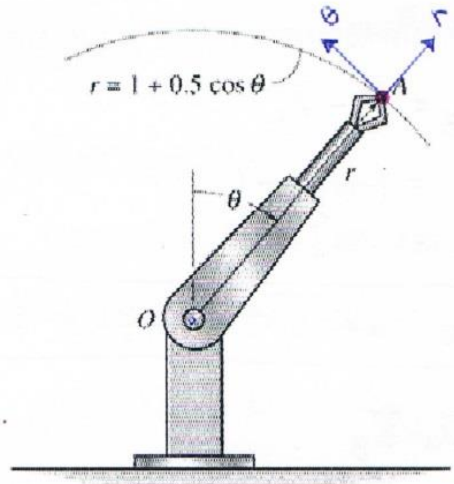
$a_r = \ddot{r} - r\dot{\theta}^2 = 4500\sec\theta(2\tan^2\theta + 1) - 500\sec\theta(3)^2 = 9000\sec\theta\tan^2\theta$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(1500\sec\theta\tan\theta)(3) = 9000\sec\theta\tan\theta$

$a = 9000\sqrt{\sec^2\theta\tan^4\theta + \sec^2\theta\tan^2\theta}$

$a = 9000\sec^2\theta\tan\theta$ mm/s^2 **Ans.**

154. เนื่องจากการยืด/หดได้ของปลายแขนหุ่นยนต์แขนยืดไปตามเส้นทาง limacon ($r = 1 + 0.5\cos\theta$ m) ที่ตำแหน่ง $\theta = \pi/4$ แขนมีความเร็วเชิงมุม 0.6 rad/s และมีความเร่งเชิงมุม 0.25 rad/s^2 จงหาองค์ประกอบเชิงรัศมี (radial) และเชิงขวาง (transverse) ของความเร็วและความเร่ง ของวัตถุ A ที่ถูกจับอยู่ที่มือจับ



$\theta = \pi/4$

$\dot{\theta} = 0.6$ rad/s

$\ddot{\theta} = 0.25$ rad/s^2

$\frac{dr}{dt}$

$r = 1 + 0.5\cos\theta$ m $= 1.3535$ m

$\dot{r} = (-0.5\sin\theta)\dot{\theta} = -0.2121$ m/s

$\ddot{r} = -0.5\left[\sin\theta \cdot \ddot{\theta} + \dot{\theta}(\cos\theta)\dot{\theta}\right]$

$= -0.5\left[\ddot{\theta}\sin\theta + (\dot{\theta})^2\cos\theta\right]$

$= -0.2157$ m/s^2

$v_r = \dot{r} = -0.5(\sin\pi/4)\,0.6 = \boxed{-0.2121 \text{ m/s}}$ #

$v_\theta = r\dot{\theta} = (1 + 0.5\cos\frac{\pi}{4})\,0.6 = \boxed{0.8121 \text{ m/s}}$ #

$a_r = \ddot{r} - r(\dot{\theta})^2 = -0.2157 - 1.3535(0.6)^2 = \boxed{-0.703 \text{ m/s}^2}$ #

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.3535(0.25) + 2(-0.2121)0.6$

$\boxed{= 0.084 \text{ m/s}^2 \ \#}$

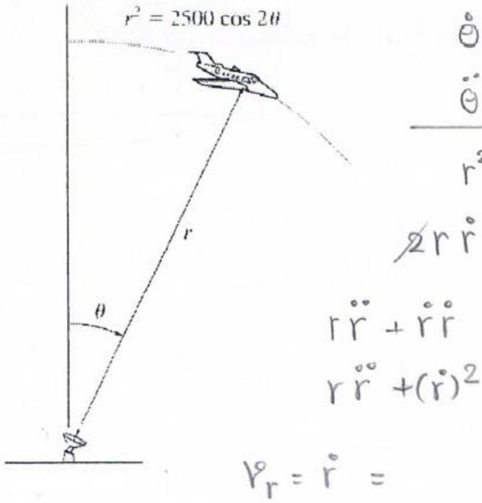
12–194. For a short time the jet plane moves along a path in the shape of a lemniscate, $r^2 = (2500\cos 2\theta)$ km^2. At the instant $\theta = 30°$, the radar tracking device is rotating at $\dot{\theta} = 5(10^{-3})$ rad/s with $\ddot{\theta} = 2(10^{-3})$ rad/s^2. Determine the radial and transverse components of velocity and acceleration of the plane at this instant.

v_r

v_θ

a_r

a_θ



สนใจขณะนี้

$\theta = 30°$

$\dot{\theta} = 5 \times 10^{-3}$ rad/s

$\ddot{\theta} = 2 \times 10^{-3}$ rad/s^2

$r^2 = 2500\cos 2\theta \rightarrow r = 35.36$ km

$\cancel{2} r\dot{r} = -2500\sin 2\theta\,(\cancel{2})\,\dot{\theta} \rightarrow \dot{r} = -0.306$ km/s

$r\ddot{r} + \dot{r}\dot{r} = -2500\left[\sin 2\theta \cdot \ddot{\theta} + \dot{\theta}(\cos 2\theta)\cdot 2\dot{\theta}\right]$

$r\ddot{r} + (\dot{r})^2 = -2500\left[(\sin 60°)(2\times10^{-3}) + (\cos 60°)(2)(5\times10^{-3})^2\right]$

$\rightarrow \ddot{r} =$

$v_r = \dot{r} =$

$v_\theta = r\dot{\theta} =$

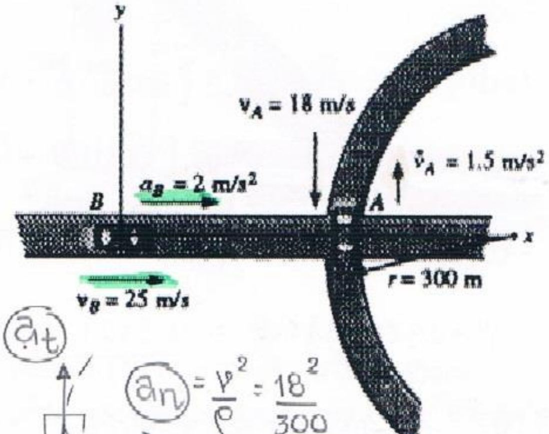
$a_r = \ddot{r} - r(\dot{\theta})^2 =$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} =$

Drill : Relative...

ความหน่วง

12 - 31. The driver of car B observes the motion of the car at A. At the instant shown, car A has a speed of 18 m/s and is reducing its speed at the rate of 1.5 m/s^2. Car B is accelerating at 2 m/s^2 and has a speed of 25 m/s. Determine the magnitudes of the velocity and acceleration of A with respect to B. Car B is moving along a curve having a radius of $r = 300$ m.



a_t

$a_n = \frac{v^2}{\rho} = \frac{18^2}{300}$

$= 1.08$

n

t

$a_t \to 1.5\bar{j}$

$v_{A|B}$, $a_{A|B}$

$\vec{v}_{A|B} = \vec{v}_A - \vec{v}_B$

$= (-18\bar{j}) - (25\bar{i})$

$= -25\bar{i} - 18\bar{j}$

$\therefore \boxed{v_{A|B} = \sqrt{25^2 + 18^2} = 30.81 \text{ m/s}}$

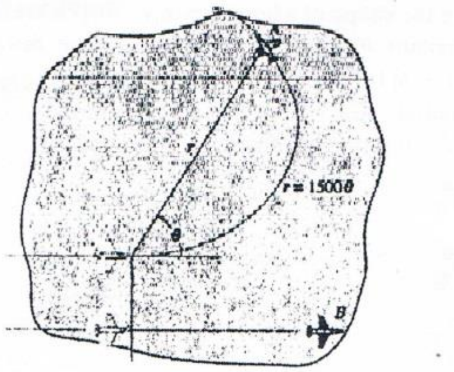
$\vec{a}_{A|B} = \vec{a}_A - \vec{a}_B$

$= (1.5\bar{j} + 1.08\bar{i}) - (2\bar{i})$

$= -0.92\bar{i} + 1.5\bar{j}$

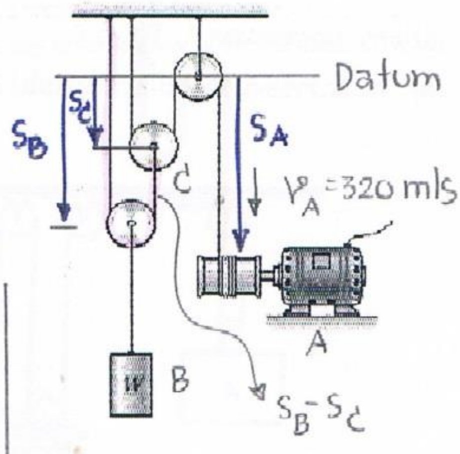
$\therefore \boxed{a_{A|B} = \sqrt{0.92^2 + 1.5^2} = 1.76 \text{ m/s}^2}$

***12-200.** Two planes A and B are flying side by side at a constant speed of 900 km/h. Maintaining this speed, plane A begins to travel along the spiral path $r = (1500\theta)$ km, where θ is in radians, whereas plane B continues to fly in a straight line. Determine the speed of plane A with respect to plane B when $r = 750$ km.



Drill : Constrained motion..

จงหาความสูง (h) ของน้ำหนัก W ที่ถูกยกขึ้นใน ช่วงเวลา 5 s. ถ้าดรัม(drum)ของระบบยกของ หมุนพันสายเคเบิลในอัตราคงที่ 320 m/s



$S_A + 2S_C = l_1$

$S_B + (S_B - S_C) = l_2 \rightarrow 2S_B - S_C = l_2$

$v_A + 2v_C = 0$ —①

$2v_B - v_C = 0$ —②

$2\times② + ①;\quad 4v_B + v_A = 0$

$4v_B + 320 = 0$

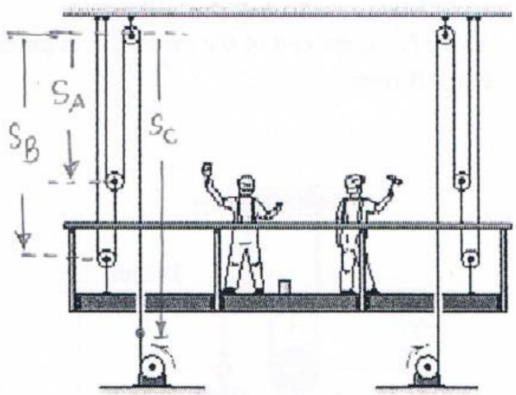
$v_B = -80$ m/s $= 80$ m/s ↑

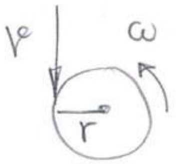
$S = vt$

$h = 80(5)$

$\boxed{h = 400 \text{ m}}$ #

2.60 เครื่องกว้านวางอยู่บนนั่งร้าน สำหรับใช้ยกนั่งร้านขึ้นหรือลง การหมุนในทิศดังแสดงในรูป นั่งร้านกำลังถูกดึงขึ้น โดยดรัม(drum)แต่ละตัว มีเส้นผ่านศูนย์กลางเท่ากับ 200 mm และหมุนที่อัตรา 40 รอบต่อนาที จงหาความเร็วขึ้นของนั่งร้าน





$v = \omega r$

$v = (2\pi f)r$

$= \left(2\pi \times \frac{40}{60}\right)\frac{100}{1000}$

$v_C = 0.4189$ m/s

$2S_A + S_C = L_1 \rightarrow 2v_A + v_C = 0$ —①

$S_B + (S_B - S_A) = L_2 \rightarrow 2v_B - v_A = 0$ —②

$2\times② + ①;\quad 4v_B + v_C = 0$

$4v_B = -0.4189$

$v_B = -0.105$ m/s

$\therefore \boxed{v_B = 0.105 \text{ m/s} \uparrow}$

1.4 มวล A เคลื่อนที่ลงจากหยุดนิ่งด้วยความเร่งคงที่ ถ้ามวล A มีความเร็ว 27 cm/s เมื่อเวลาผ่านไป 7 s จงหา

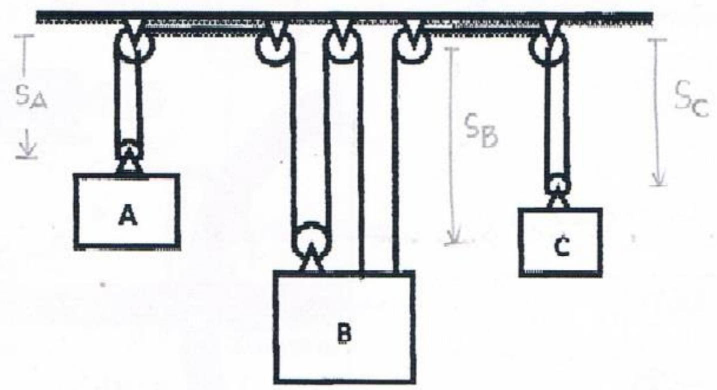
a) ความเร่งของมวล A, B และ C

b) ความเร็วของมวล B เมื่อเวลาผ่านไป 2 s

b) $v = v_0 + a t$

$= 0 + 7.72(2)$

$v = 15.44$ m/s



A: $v = v_0 + at$

$27 = 0 + a_A(7)$

$a_A = \frac{27}{7}$ m/s^2

$= 3.86$ m/s^2 ↓

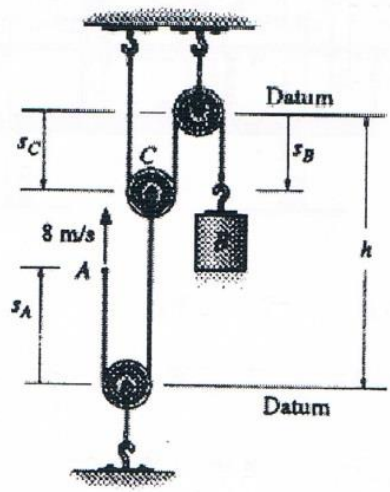
$2S_A + 3S_B = l_1 \rightarrow 2a_A + a_B = 0$ — ①

$S_B + 2S_C = l_2 \rightarrow a_B + 2a_C = 0$ — ②

จาก ① ; $a_B = -2a_A = -7.72$ ↑

จาก ② ; $-7.72 = -2a_C \rightarrow a_C = 3.86$ m/s^2 ↓

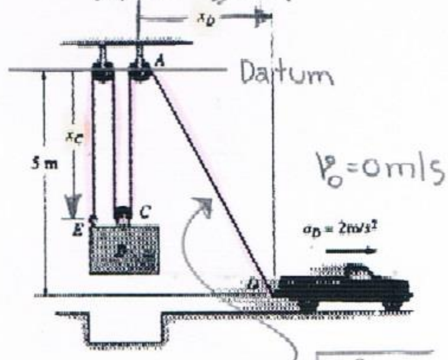
12 - 27. If the end of the cable at *A* is pulled upwards with a speed of 8 m/s, determine the speed at which block *B* rises.



$2S_C + S_B = l_1$ — ①

$S_A + (h - S_C) = l_2$ — ②

12 - 28. The cable is attached to the block B at E, wraps around three pulleys, and is tied to the back of a truck. If the truck starts from rest when x_D is zero, and moves forward with a constant acceleration of $a_D = 2\ \text{m/s}^2$, determine the speed of block B at the instant $x_D = 3\text{m}$. Neglect the size of the pulleys in the calculation. When $x_D = 0$, $x_B = 5$ m so that points C and D are at the same elevation.



$$3x_C + \sqrt{x_D^2+25} = l \quad \text{—(1)}$$

$$\frac{d(1)}{dt}, \quad 3v_C + \frac{\cancel{2}x_D v_D}{\cancel{2}\sqrt{x_D^2+25}} = 0$$

$$\frac{d\sqrt{\square}}{dt} = \frac{\square'}{2\sqrt{\square}}$$

$$v^2 = v_0^2 + 2a(s - s_0)$$

$$v^2 = 0 + 2(2)(3)$$

$$v = \sqrt{12} = 2\sqrt{3}\ \text{m/s}$$

$$\cancel{3}v_B + \frac{\cancel{3}(2\sqrt{3})}{\sqrt{3^2+25}} = 0$$

$$v_B = -0.594\ \text{m/s}$$

$$\boxed{v_B = 0.594\ \text{m/s}\ \uparrow}$$